Course Syllabus for
DAUSY National Ph.D. Program in Autonomous Systems
(year 2023-24)

Course title	From Least Squares to Subspace Identification
Scientific Discipline Sector	ING-INF/04
Hours of instruction	20 hours
CFU	2 CFU
Semester, period	Second Semester (February-March) 2024
Goal	This course aims at providing PhD students an introductory overview of some classical and modern tools from the fields of system identification and data analysis. Starting from the well-known theory of ordinary least squares, interpreted as a subspace projection problem, the course will discuss general data fitting methodologies and applications to linear system identification, such as the estimation of dynamical models via prediction error methods. Then, interpreting linear system identification as a subspace projection problem, the so-called subspace identification methodologies will be introduced, starting from the seminal Ho-Kalman method and towards the MOESP algorithm. The final goal is to provide PhD students with the necessary background for starting research in the field of data analysis and modern system identification. Each lesson consists in lectures, numerical examples, simulation (MATLAB).
Syllabus	- Basics of linear algebra: vectors, matrices, subspaces. Four fundamental subspaces of a matrix, pseudo-inverses, projections. Singular Value Decomposition and its relationship with pseudo-inverses and the four fundamental subspaces. QR factorization. First applications to data analysis and data reduction. - Approximate and least-norm solutions to overdetermined and underdetermined linear systems of equations via ordinary least squares. Geometric interpretation of least squares. Applications to data fitting. - Basic facts on estimation: real phenomena and their models, data collection, training and testing datasets, model validation. - Short review of linear dynamical systems: time-invariant models and their solutions. Impulse response and transfer functions. Structural properties (observability, controllability), stability. - Classical results in system identification: least-squares solutions of prediction error methods for simple polynomial models (ARX models). - Basics of subspace identification: the Ho-Kalman method for deterministic realization of impulse responses. Ho-Kalman revisited (general inputs, measurement noise). Subspace methods: the MOESP algorithm for noisy systems. Model order reduction via truncated realization.

Bibliography	Recommended books: - Verhaegen, M., & Verdult, V. (2007). *Filtering and system identification: a least squares approach*. Cambridge University Press. - Van Overschee, P., & De Moor, B. (2012). *Subspace identification for linear systems: Theory—Implementation—Applications*. Springer Science & Business Media. Notes and supporting material from lecturer.
Examination method	Final examination by written test on theory and applications.