Course Syllabus for
DAUSY National Ph.D. Program in Autonomous Systems
(year 2022-23)

Course title	Linear Algebra for Control Applications
Scientific Discipline Sector	ING-INF/04
Hours of instruction	20 hours
CFU	2 CFU
Semester, period	Second semester, february-march 2023
Goal	The course will introduce advanced linear algebra tools that are commonly used in many applications in Control and System Theory. The course will address this topic from different perspective: (i) Theory with formal proofs of many results, (ii) Algorithms to understand the most common algorithms used in MATLAB or Python for Linear Algebra, (iii) Implementation via MATLAB of algorithms and performance evaluation on large data sets.
Syllabus	• Vectors: inner products, norms, main operations (average, standard deviation, …) • Matrices: matrix-vector and matrix-matrix multiplication, Frobenius norm, • Complexity, sparsity • Special matrices: Diagonal, Upper Triangular, Lower triangular, Permutation (general pair), inverse and orthogonal • A square and invertible: LU decomposition (aka gaussian elimination), LU-P decomposition, Cholesky decomposition • Ax=b via LU-P decomposition: forward and backward substitution • (sub)Vector spaces: definitions, span, bases (standard, orthogonal, orthonormal), dimension, direct sum, orthogonal complement, null space, orthogonal complement theorem • Gram-Smith orthogonalization and QR decomposition (square and invertible A, general non-square) • Ax=b via QR decomposition. LU-P vs QR • Linear maps: image space, kernel, column and row rank

	• Fundamental Theorem of Linear Algebra (Part I): rank-nullity Theorem, the 4 fundamental subspaces • Eigenvalues/eigenvector and Shur decomposition • Projection matrices: oblique and orthogonal, properties • Positive semidefinite matrices: properties and quadratic functions, square root matrix • Properties of A'A and AA' and Polar decomposition • Singular Value Decomposition: proofs and properties • Pseudo-inverse: definition and relation to SVD • Fundamental Theorem of Linear Algebra (Part II): special orthogonal basis for diagonalization • Least-Squares: definition, solution and algorithms • Ill-conditioned problems vs stability of algorithms, numerical conditioning • Regularized vs truncated Least-Squares
Bibliography	• S. Boyd, L. Vanderberghe, "Introduction to Applied Linear Algebra", Cambridge University Press, 2018, http://vmls-book.stanford.edu/ • G. Strang, "The Fundamental Theorem of Linear Algebra", The American Mathematical Monthly, vol. 100(9), pp. 848-855, 1993, https://www.jstor.org/stable/2324660 • G. Strang, "Linear Algebra and Learning From Data", Wellesley - Cambridge Press, 2019
Examination method	Homework sets assigned after each lecture. Final examination by written test on theory and algorithms.