Title: Mathematical introduction to control and optimal control problems

Content: In this course, of around 25 academic hours plus some seminars given by other experts, the main mathematical theory and tools will be introduced for the study of control and optimal control problems. In particular, the course will focus on controllability and stability conditions for linear and nonlinear systems; on necessary conditions for optimality both for linear/quadratic and nonlinear optimal control problems; on the dynamic programming method and the Bellman equations for optimal control problems as well as for differential games; on the geometric control point of view and Lie groups. Below there is a list of possible arguments.
The major prerequisites are: linear algebra, differential and integral calculus in one and several variables, Cauchy problem for systems of ordinary differential equations, uniform and integral convergence of functions, basic notions of metric spaces.

a) Introduction to control and optimal control: presentation of the models and the problems, with examples from the applications. (1 hour)

b) Recalls on Ordinary Differential Equations with measurable time-dependent dynamics: existence
and uniqueness and basic estimates on the trajectories. (1 hour)

c) Recalls on Partial Differential Equations: functional spaces and weak formulations. (1 hours)

d) Kalman condition for the controllability of the linear systems (2 hours)

e) Stability for linear systems (2 hours)

f) Pontryagin's theorem (3 hours)

g) Linear systems with quadratic costs (2 hours)

h) Hamilton-Jacobi-Bellman equation for the optimal control and viscosity solutions. (2 hours)

i) Differential games and the Isaacs equation. (2 hours)

l) Problems with many agents. (2 hours)

m) Introduction to Lie groups (1 hour)

n) Driftless affine systems: Lie brackets, chow conditions and orbit theorem (2 hours)

o) Left invariant optimal control problems on the group of motions of plane and on SO(3) and SO(2). (2-3 hours)

p) Seminar by experts

The course will be held in **hybrid** modality.

References:

[1] A.A.Agrachev and Y.L.Sachkov: Control theory from the geometric viewpoint, volume 87 of Encyclopedia of Mathematical Sciences. Springer-Verlag, Berlin, 2004, Control Theory and Optimization II.

[2] F. Bagagiolo: Notes of a doctoral course 2020, https://drive.google.com/file/d/1H0CdKQseE4wBIxGjUQnz_KeDx6kV_TnV/view?usp=sharing%22%22

[3] M. Bardi and I. Capuzzo Dolcetta: Optimal Control and Viscosity Solutions of Hamilton-
Jacobi-Bellman Equations, Birkh auser, 1997.

[4] U. Boscain and M. Sigalotti: Introduction to controllability of non-linear systems. Free on the web.

[5] A. Bressan: Noncooperative Differential Games, Lecture Notes, 2010. Free on the web

[6] A. Bressan: Viscosity Solutions of Hamilton-Jacobi Equations and Optimal Control
Problems, Lecture Notes, 2011. Free on the web

[7] P. Cardaliaguet: Notes on Mean Field Games, Lecture Notes, 2013. Free on the web.

[8] F. Jean: Control of nonholonomic systems: from sub-Riemannian geometry to motion planning. SpringerBriefs in Mathematics, Springer, Cham, 2014.

[9] V. Jurdjevic: Geometric control theory, Cambridge University Press.

[10] J. Macki and A. Strauss: Introduction to Optimal Control Theory, Springer, 1995.

[11] M.J. Osborne and A. Rubinstein: A Course in Game Theory, MIT Press, 1994.